Tubular Surface Evolutions for Segmentation of Tubular Structures With Applications to the Cingulum Bundle From DW-MRI

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Joint Work With ...

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Outline

Introduction to Fiber Bundle Segmentation and Motivation
  Cingulum Bundle ... A Structure of Importance
  Methods for DW-MRI Fiber Bundle Analysis

Energy Models for Extracting Tubular Fiber Bundles
  A Tubular Model for the Cingulum Bundle
  Constructing Energies on Tubular Surfaces

Optimization of Tubular Energies: Sobolev Gradient Flows

Experimental Result

Summary
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Summary
The Cingulum Bundle and Other Fiber Pathways

- 5-7mm in diameter fiber bundle: interconnects limbic system
  - fibers are mostly parallel, sometimes intersecting
- forms a “ring-like belt” around the corpus callosum
- Involved with executive control and emotional processing
- May be linked to schizophrenia
Imaging the Cingulum Bundle in the Brain: DW-MRI

\[ I : \mathbb{R}^3 \times S^2 \rightarrow \mathbb{R}^+ \]

direction of magnetic field

spatial locations

image intensity

[0,0,0]^T [1,1,0]^T [0,1,1]^T [1,0,1]^T [0,1,-1]^T [1,-1,0]^T [1,0,1]^T
Imaging the Cingulum Bundle in the Brain: DW-MRI

We show visualization of DW-MRI and the cingulum bundle: Movie.
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Summary
DW-MRI for Structural and Connectivity Information

- DW-MRI Data
- Tractography & Segmentation Algorithms
- Connectivity Information
- Structural Information
Overview of Our Approach for Fiber Bundle Analysis

- **DW-MRI Data**
  - Streamline Tractography
  - Geodesic Tractography

- **Tracts or Streamlines**

- **Optimal Path**
  - Volumetric Segmentation

- **Fiber Bundles**
  - Clustering

- **Structural Information**

- **Connectivity Information**

**Tractography & Segmentation Algorithms**

**Introductions**
- Fiber Bundle Segmentation and Motivation
- Energy Models for Extracting Tubular Fiber Bundles
- Optimization of Tubular Energies: Sobolev Gradient Flows
- Experimental Result

**Methods for DW-MRI Fiber Bundle Analysis**

**Cingulum Bundle ... A Structure of Importance**

**Summary**

Sundaramoorthi et al.
Tubular Segmentation of Cingulum Bundle
Our Approach: Geodesic Tractography

Detecting A Single Fiber (Melonakos et al., IEEE PAMI 2008)

Given two seed points, find \textit{optimal path} between them.

Let $c : [0, 1] \rightarrow \mathbb{R}^3$

$$E(c) = \int_c \psi(c(s), c_s(s)) \, ds$$

Based on DW-MRI
Our Approach: Geodesic Tractography

**Detecting A Single Fiber** (Melonakos et al., IEEE PAMI 2008)

Given two seed points, find *optimal path* between them.

Let $c : [0, 1] \rightarrow \mathbb{R}^3$

\[ E(c) = \int_c \psi(c(s), c_s(s))\, ds \]

Based on DW-MRI

Position

Arclength

Tangent

Globally minimize: fast sweeping (Kao et al. 2003) for some $\psi$
Our Approach: Geodesic Tractography
Our Approach: Geodesic Tractography
Our Approach: Geodesic Tractography
Our Approach: Volumetric Segmentation Method

Volumetric Surface Methods Applied to DW-MRI

Surface Obtained From DT-MRI

- DTI Volumetric Segmentation:
  - Region-Based Methods (e.g. Lenglet et al., Wang and Vemuri)
  - Edge-Based Method (Melonakos et al.)
- We Tailor Above Methods to Fiber Bundles
  - Shape Prior Needed
  - Challenge: Non-homogeneity of statistics of the cingulum bundle
Non-Uniform Statistics of Cingulum Bundle

Sagittal Slice of DT-MRI of a Brain

CB = Cingulum bundle
CC = Corpus Callosum
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Modeling the Cingulum Bundle as a Tubular Surface

Why Model the Cingulum Bundle as a Tubular Surface?

- **Natural Shape Prior:**
  - Cingulum Bundle is approximately tubular
  - DW-MRI is noisy and filled with irrelevant features; cingulum bundle hard to segment without prior

- **Significant Dimension Reduction:**
  - Segmentation reduced from detecting a *surface* to a *curve*

- **Statistical Shape Analysis of Tubular Surfaces is Easier**
  - Main point of segmenting the cingulum bundle: *population studies*, where statistical analysis must be performed to compare controls and disease cases
Modeling the Cingulum Bundle as a Tubular Surface

- **Given** center-line: \( c : [0, 1] \rightarrow \mathbb{R}^3 \), and radius function: \( r : [0, 1] \rightarrow \mathbb{R}^+ \)
- **Define** the tubular surface, \( S : S^1 \times [0, 1] \rightarrow \mathbb{R}^3 \), as

\[
S(\theta, u) = c(u) + r(u)[n_1(u) \cos \theta + n_2(u) \sin \theta]
\]

where \( n_1, n_2 : [0, 1] \rightarrow \mathbb{R}^3 \) are normals to the curve \( c \): orthonormal, smooth, and \( c'(u) \cdot n_i(u) = 0 \)

- **Tubular Surface Identified With a 4-D Curve**: \( S \leftrightarrow \tilde{c} = (c, r) \in \mathbb{R}^4 \)
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**Segmentation Algorithm: Variational Approach**

- Formulate energies on 4D curves, $\tilde{c}$, $(S \leftrightarrow \tilde{c})$
- *Weighted length* energies:

$$E(\tilde{c}) = \int_{\tilde{c}} \Psi(\tilde{c}(\tilde{s}), \frac{c'(\tilde{s})}{\|c'(\tilde{s})\|}) \, d\tilde{s}, \quad \tilde{c} = (c, r)$$

**Diagram:**
- **Position & Radius**
- **Tangent of Centerline**
- **Local Cost**
- **4D curve arclength**
Segmentation Algorithm: Variational Approach

- Formulate energies on 4D curves, \( \tilde{c} \),
  \( (S \leftrightarrow \tilde{c}) \)
- \textit{Weighted length} energies:

\[
E(\tilde{c}) = \int_{\tilde{c}} \psi(\tilde{c}(\tilde{s}), \frac{c'(\tilde{s})}{|c'(\tilde{s})|}) \, d\tilde{s}, \quad \tilde{c} = (c, r)
\]

- \( \psi(\tilde{p} = (p, r), v) \) to incorporate statistics of DW-MRI \textit{local} to \( \tilde{p}, v \)
- Rather than one set of global statistics
Segmentation Algorithm: Variational Approach

Example 1: Choice of $\Psi$

- Let $I$ be DW-MRI
- $\Psi_1$ minimized when mean of $I$ inside disc, $\mu_{D(p,r,v)}$, maximally different from mean of $I$ outside, e.g.

$$\psi_1(\tilde{p}, v) = \frac{1}{1 + \| \mu_{D(\tilde{p}, v)} - \mu_{D((p, r), v)} \|_{D(\tilde{p}, v)}^2}$$
Segmentation Algorithm: Variational Approach

**Example 1: Choice of $\Psi$**

- Let $I$ be DW-MRI
- $\Psi_1$ minimized when mean of $I$ inside disc, $\mu_D(p,r,v)$, *maximally* different from mean of $I$ outside, e.g.

$$\Psi_1(\tilde{p}, v) = \frac{1}{1 + \|\mu_D(\tilde{p}, v) - \mu_D((p,\alpha r), v)\|_{D(\tilde{p}, v)}^2}$$

*Need to define mean and norm for pixel-wise DW-MRI data*
Segmentation Algorithm: Variational Approach

**Defining Mean and Norm for DW-MRI** (Easier than DT-MRI)

- **DW-MRI**: $I : \mathbb{R}^3 \times S^2 \rightarrow \mathbb{R}^+$
- DW-MRI sampled uniformly directionally at each pixel, $p \in \mathbb{R}^3$
- Addition: add *corresponding* values at directions

$$I(p_1, \cdot) \quad I(p_2, \cdot)$$
Segmentation Algorithm: Variational Approach

Defining Mean and Norm for DW-MRI (Easier than DT-MRI)

- DW-MRI: $I : \mathbb{R}^3 \times \mathbb{S}^2 \rightarrow \mathbb{R}^+$
- DW-MRI sampled uniformly directionally at each pixel, $p \in \mathbb{R}^3$
- Addition: add corresponding values at directions

Given $f_1, \cdot, f_n : \mathbb{S}^2 \rightarrow \mathbb{R}^+$ (DW-MRI at $n$ different spatial locations):

\[
\text{mean of } f_1, \ldots, f_n(v) := \frac{1}{n} \sum_{i=1}^{N} f_i(v), \quad \| f_i \|^2 = \int_{\mathbb{S}^2} |f_i(v)|^2 \, dS(v)
\]
A Tubular Model for the Cingulum Bundle

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Segmentation Algorithm: Variational Approach

Example 2: Choice of $\Psi$

$\Psi_2(p, r, v) = r \int_0^{2\pi} \phi(p + rv^\perp(\theta)) \, d\theta$

$v^\perp(\theta) = n_1 \cos \theta + n_2 \sin \theta$

$\phi(x) = \frac{1}{|B(x, R)|} \int_{B(x, R)} \| I(y, \cdot) - \mu_{B(x, R)}(\cdot) \|^2 \, dy$

- $B(x, R)$ is a ball
- $\phi$ is an “edge-detector”
- Corresponding energy to $\Psi_2$ is related to a weighted surface area.

Sundaramoorthi et al. Tubular Segmentation of Cingulum Bundle
Energy Optimization: Gradient Descent/Ascent

\[ E(\tilde{c}) = \int \psi(\tilde{c}(\tilde{s}), \tilde{c}_s(\tilde{s})) \, d\tilde{s} \]

- Why Gradient Ascent/Descent?
  - Global techniques (e.g. minimal paths) do not apply to direction-based energies
  - Not interested in global optimum: \( \Psi_2 \)
- Gradient flow: \( \partial_t \tilde{c} = \pm \nabla E(\tilde{c}) \)
Gradient Flow of Tubular Energy

Given the energy

\[ E(\tilde{c}) = \int_{\tilde{c}} \psi(\tilde{c}(\tilde{s}), \tilde{c}_{\tilde{s}}(\tilde{s}))\,d\tilde{s} \]

we get the following gradient flow:

\[ \tilde{c}_t = \pm (\tilde{c}_{\tilde{s}} \cdot \psi_{\bar{v}\bar{p}} - \psi_{\bar{p}}) \perp \pm (\psi + \psi_{vv})\tilde{c}_{\tilde{s}\tilde{s}} \]

2nd covariant derivative

projection onto \( \tilde{c}_{\tilde{s}} \perp \)

curvature vector

Well-posedness: \( \psi + \psi_{vv} \) must be positive definite (negative definite for ascent flow)

▶ For \( \psi_1 \) and \( \psi_2 \), this condition is NOT satisfied

▶ Flow is ILL-POSED
Gradient Flow of Tubular Energy

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Well-posedness: \( \psi + \psi_{vv} \) must be positive definite (negative definite for ascent flow)

- For \( \psi_1 \) and \( \psi_2 \), this condition is NOT satisfied
- Flow is ILL-POSED
Calculating Gradient Flows of Geometric Energies

1. Compute

\[ dE(c) \cdot h = \left. \frac{d}{dt} E(c + th) \right|_{t=0} \]

change in \( E \) in direction \( h \)

for generic \( c \) and \( h \).

c : \( S^1 \to \mathbb{R}^2 \), and \( h : S^1 \to \mathbb{R}^2 \) is a vector field on \( c \) (i.e., a perturbation or deformation of \( c \)).
Calculating Gradient Flows of Geometric Energies

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2. Manipulate \( dE(c) \cdot h \) into the form

\[ \int _c h(s) \cdot v(s) \, ds \]

where \( v \) is some perturbation of \( c \).

\( c : S^1 \rightarrow \mathbb{R}^2 \), and \( h : S^1 \rightarrow \mathbb{R}^2 \) is a

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Calculating Gradient Flows of Geometric Energies

1. Compute

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2. Manipulate $\text{d}E(c) \cdot h$ into the form

$$\int_c h(s) \cdot v(s) \, ds$$

where $v$ is some perturbation of $c$.

3. $v$ is the gradient: direction which maximizes $E$ fastest.

$c : S^1 \to \mathbb{R}^2$, and $h : S^1 \to \mathbb{R}^2$ is a vector field on $c$ (i.e., a perturbation or deformation of $c$)
Calculating Gradient Flows of Geometric Energies

1. Compute
   \[ \frac{dE(c) \cdot h}{dt} = \left. \frac{d}{dt} E(c + th) \right|_{t=0} \]
   change in \( E \) in direction \( h \)
   for generic \( c \) and \( h \).

2. Manipulate \( dE(c) \cdot h \) into the form
   \[ \int_c h(s) \cdot v(s) \, ds \]
   where \( v \) is some perturbation of \( c \).

3. \( v \) is the gradient: direction which maximizes \( E \) fastest.

4. Gradient descent flow: \( \partial_t C = -v(C) \).

\[ c : S^1 \to \mathbb{R}^2, \text{ and } h : S^1 \to \mathbb{R}^2 \text{ is a vector field on } c \text{ (i.e., a perturbation or deformation of } c) \]
Traditional Norm That Led To Ill-posed Flows: $L^2$

Norm on deformations assumed in deformable model literature:

Geometric $L^2$-type norm

\[ \| h \|_{c,L^2}^2 := \int_c |h(s)|^2 \, ds \]
Gradient Depends on *Norm* on Deformations of Curve

**Proposition**

The gradient $\nabla E(c)$ is the vector in $T_c M$ that satisfies (if $dE(c) \neq 0$)

$$
\frac{dE(c) \cdot (\nabla E(c))}{\| \nabla E(c) \|_c} = \sup_{h \in T_c M \setminus \{0\}} \frac{|dE(c) \cdot h|}{\| h \|_c}.
$$

Thus, gradient is the most *efficient* perturbation, *i.e.*, maximizes

change in energy by moving in direction $h$

over cost of moving in direction $h$
Gradient Depends on *Norm* on Deformations of Curve

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▶ Thus, gradient is the most *efficient* perturbation, *i.e.*, maximizes

\[
\frac{\text{change in energy by moving in direction } h}{\text{cost of moving in direction } h}.
\]

▶ By choosing different \( \| \cdot \|_c \) than we obtain a different path to minimize \( E \) without changing \( E \).
Proposed Norm: Sobolev-type Norm

Geometric Sobolev-type norm (open curves)

\[ \| h \|_{c,Sob}^2 = L \int_c \left| D_s h(s) \right|^2 ds \]

length of curve \quad deformation of \ c

Original Motivation for Sobolev Norms

Energy, $E$, **is not** changing; the scale that is used to measure cost (length) of a perturbation **is** changing.

**Diagram:**

- Represents local neighborhood of curve $c \in M$.
- Spatial scale is relative to “distance” in $M$ measured through $\| \cdot \|_0$ or $\| \cdot \|_1$.
- $6$ means energy $E = 6$ at particular point (curve).
Original Motivation for Sobolev Norms

Sobolev Norms Favor Coarse Scale Motions

\[ \|h\|_{c,H^0}^2 := \int_c |h(s)|^2 \, ds \]

\[ \|h\|_{c,Sob}^2 = L \int_c |D_s h(s)|^2 \, ds \]
Original Motivation: Sobolev Norms Robust to Noise and Local Minima

**Region-Based Segmentation:** \( E = \text{Chan-Vese energy, TIP 2001} \)

\[-\nabla_{L^2} E + \alpha \kappa \mathcal{N} \]
Original Motivation: Sobolev Norms Robust to Noise and Local Minima

**Region-Based Segmentation:** \((E = \text{Chan-Vese energy}, \text{TIP 2001})\)

\[-\nabla L^2 E + \alpha \kappa N\]

\[-\nabla L^2 E + \beta \kappa N\]

\((\beta >> \alpha)\)
Original Motivation: Sobolev Norms Robust to Noise and Local Minima

Region-Based Segmentation: \( E = \text{Chan-Vese energy, TIP 2001} \)

\[
-\nabla_{L^2} E + \alpha \kappa N
\]

\[
-\nabla_{L^2} E + \beta \kappa N
\]
\[ (\beta \gg \alpha) \]

\[
-\nabla_{Sob} E
\]
Sobolev Stabilizes $\mathbb{L}^2$ Flows Involving Length

Let $E$ be an energy, one can show that

$$\nabla_{\text{Sob}} E = K \ast \nabla_{\mathbb{L}^2} E$$

**important property:**

$$K''(s) = \frac{1}{L^2} \left( \frac{1}{L} - \delta(s) \right)$$

If $E(c) = \text{length of } c$, then $\nabla_{\mathbb{L}^2} E(c) = -c_{ss}$ and

$$\nabla_{\text{Sob}} E(c) = K \ast (-c_{ss}) = \frac{c - \overline{c}}{L}$$

($\overline{c}$ is the centroid of $c$).
Fiber Bundle Extraction: Two Step Approach

Given: Two seed regions (beginning and end of fiber bundle)

1. Find an open curve, \( c_{init} : [0, 1] \rightarrow \mathbb{R}^3 \) in fiber bundle.
   - Rough estimation of a curve in bundle required (e.g. geodesic tractography)
Fiber Bundle Extraction: Two Step Approach

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2. Initialize tubular surface: $\tilde{c}(0) = (c(0), r(0)) = (c_{init}, 1)$
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2. Initialize tubular surface: $\tilde{c}(0) = (c(0), r(0)) = (c_{init}, 1)$

3. Optimize tubular surface energy
Tubular Energy Optimization Procedure

Iterate:

1. Evolve interior of 4D curve with fixed endpoints

\[ \tilde{c}_t = \pm \nabla_{\text{Sob}} E(\tilde{c}) \]

\[ = \pm K(\psi_\tilde{p}) \pm \partial_{\tilde{s}} K(\tilde{\psi}_v \sqrt{1 + (r_{\tilde{s}}/|c_{\tilde{s}}|)^2} + \psi \tilde{c}_{\tilde{s}}), \]
Tubular Energy Optimization Procedure

Iterate:

1. Evolve interior of 4D curve with fixed endpoints

\[
\tilde{c}_t = \pm \nabla_{\text{Sob}} E(\tilde{c}) = \pm K(\psi_\tilde{p}) \pm \partial_s K(\psi \sqrt{1 + (r_\tilde{s}/|c_\tilde{s}|)^2} + \psi \tilde{c}_s),
\]

2. Evolve the endpoints of 4D curve (valid for e.g. \(\psi_2\))

\[
\tilde{c}_t(0) = \mp \psi \sqrt{1 + \left(\frac{r_\tilde{s}}{|c_\tilde{s}|}\right)^2} \mp \psi \tilde{c}_s
\]
\[
\tilde{c}_t(1) = \pm \psi \sqrt{1 + \left(\frac{r_\tilde{s}}{|c_\tilde{s}|}\right)^2} \pm \psi \tilde{c}_s
\]
Result of the Segmentation Method

3D Views of Result
Some Results of the Segmentation Method

A Slice-Wise View of Result
Method for fiber bundle extraction:
- E.g. Geodesic tractography for single fiber
- Initialization for volumetric segmentation

Modeled certain fiber bundles as tubular region

Future Work: Statistical analysis of cingulum bundles and functions defined on the cingulum bundle
- Tubular model makes it easy
Thank you.